

## A QUARTER CAR SIMULATION MODEL FOR VERTICAL DYNAMICAL RESPONSE ANALYSIS

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### ABSTRACT

The dynamics of a vehicle and its mechanistic understanding from the riding comfort and stability stand point of view has always been the critical area of research for dynamicists and manufacturers in the automotive industry. The dynamics is significantly influenced by disturbances, essentially unwanted vibrations induced due to kinematic excitations from numerous sources such as road terrain irregularities, aerodynamic forces and vibrations from mechanical assemblies. Designing a suspension system for optimal performance, preventing these disturbances to affect the passenger comfort while increasing riding capabilities and ensuring a smooth drive under different conditions, necessitates development of powerful, 'self-formulating' [1] computer models capable of numerically simulating a broad range of vehicular configurations and various conditions of real driving scenarios for response predictions. The paper delineates development of one such simulative ride model of a quarter of the vehicle and its subsequent implementation in MATLAB/Simulink for predicting dynamical responses to a variety of input excitations. The analytical qualification followed by parametric studies revealed the proposed model to be sensitive enough to vehicle configurations and suspension features, over and above the driving speed and road terrain roughness. The model, with appropriate treatment could be employed to help dynamicists and designers diagnose, analyze and optimize the critical dynamical attributes right at the design stage, eliminating the need of expensive prototype building and time consuming testing procedures.

**KEYWORDS:** Dynamics, Modeling, Simulation

### INTRODUCTION

Vehicle dynamics influencing ride quality and stability during maneuvers over rough terrains has flourished in the last few decades as one of the most crucial criteria [2, 3] for the subjective evaluation of vehicle performance and its mechanistic understanding has become essential for the automotive researchers and dynamicists.

One of the approaches for understanding the underlying concepts of vehicle dynamics relies on experiments and field tests, which of course is expensive but indispensable [4] for validating the computer models.

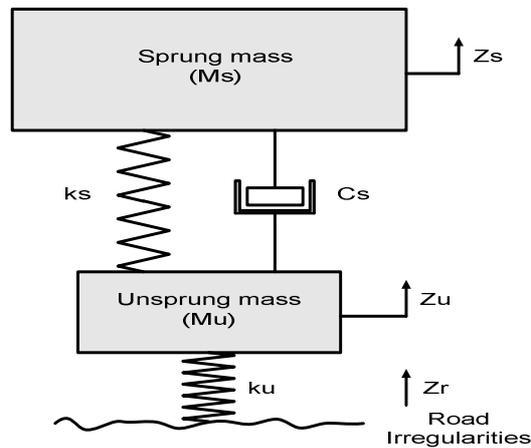
The low cost analytical one, on the other hand, relies on establishment of accurate mathematical models of individual components of a vehicle that can be integrated into comprehensive models of overall vehicle, allowing simulations and evaluation of its behavior even before being rendered in the hardware [5]. The predictive capabilities further help the dynamicists assess the importance of specific properties/features on the simulated behavior and thus serve as a means of improving performance.

## MODELING

The traditional approach to the development of computer-based formulations for vehicle system analysis involves [1] derivation of a representative mathematical model and the governing systems of equations, computer simulations and experimental validation.

The mathematical model of a vehicle for dynamical studies needs to be accurate enough in depicting its dynamic characteristics, at least, those of vital importance [6]. Commonly used vehicle models include quarter-, half- and full-vehicle models in the increasing order of complexities [4].

One such quarter-car 2DOF model on the basis of the concept of a four-wheel independent suspension [7] is considered here for simulating the vertical ride dynamics of a vehicle while travelling on a variety of road terrains. The model could be thought of representing the dynamics of a quarter of the vehicle [8, 9], e.g. the front left or front right corner of the vehicle. Such quarter models have been used predominantly for conceptual studies in the literature due to their simplicity, creditability and for being computationally efficient [10].



**Figure 1: 2DOF Quarter Vehicle Model**

The model, as illustrated in Figure 1, consists of a sprung mass ( $M_s$ ) supported by a primary suspension, which in turn is connected to the unsprung mass ( $M_u$ ). The tire interacting with road profile ( $Z_r$ ) is represented as a simple spring with stiffness ( $k_u$ ), although a damper is often included to represent the small amount of damping [11] inherent to the viscoelastic nature of the tire. The suspension unit consists of a spring and a damper with constant coefficients ( $k_s$ ) and ( $C_s$ ) respectively. The constitutive behavior of these elements is treated to be linear [12, 13]. The dynamical differential equations built via a standard force balance for sprung and unsprung mass are given as;

$$M_s \ddot{Z}_s - k_s (Z_u - Z_s) - C_s (\dot{Z}_u - \dot{Z}_s) = 0. \quad (1)$$

$$M_u \ddot{Z}_u + k_s (Z_u - Z_s) + C_s (\dot{Z}_u - \dot{Z}_s) - k_u (Z_r - Z_u) = 0. \quad (2)$$

### Simulation of Vehicle and Road Interactions

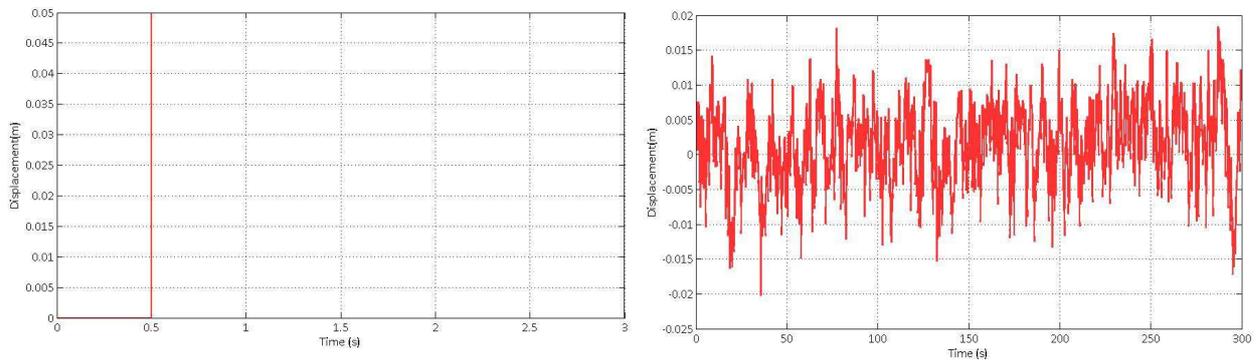
For dynamical investigations, the model, as derived above, was implemented in the MATLAB/Simulink environment by considering a test vehicle with standard suspension. All relevant parameters, other than the ones provided

by the manufacturer, to aid in simulations were estimated experimentally [14]. The parametric values for simulation pertaining to the front right corner of the test vehicle, abbreviated as QFRCo henceforth are as listed in Table 1.

**Table 1: QFRCo: Parametric Values for Simulation**

| <b>Sprung Mass</b>             | <b>320 kg</b> |
|--------------------------------|---------------|
| Unsprung mass                  | 37.5kg        |
| Suspension spring stiffness    | 22500 N/m     |
| Suspension damping coefficient | 1500 Ns/m     |

To investigate the dynamical responses, the vehicle models and the road disturbances to be encountered were simulated in the software MATLAB Simulink. The road disturbances to be fed as the input to the vehicle were modeled both as discrete and random [14], as shown in Figure 2.



**Figure 2: Input Road Disturbances Modeled as Discrete and Random Events**

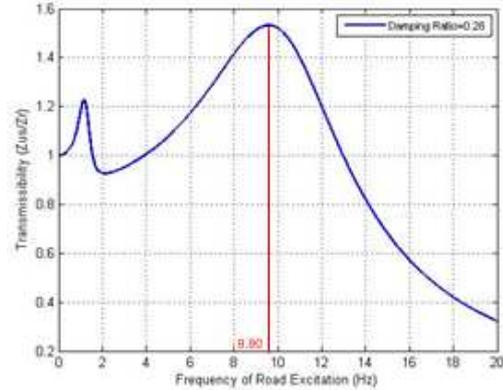
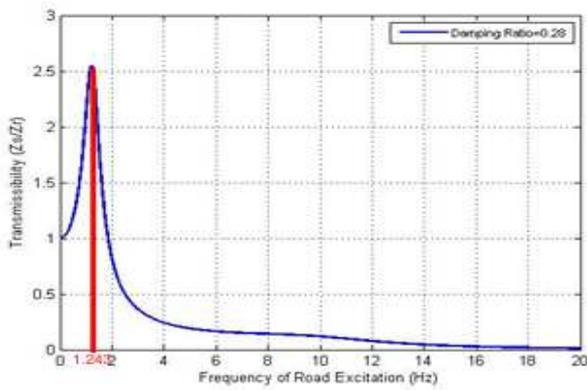
**Responses to Step Input and Validation**

The dynamical responses to a step input in the form of plots of performance evaluating parameters such as displacements, accelerations, suspension travel and tire deformations [14] were plotted. The simulative transmissibility plots [14] generated in the low frequency range 0.5-20 Hz for sprung and unsprung mass, as depicted in Figure 3 and Figure 4 respectively, exhibited two peaks corresponding to dominant frequencies of 1.22 Hz and 9.64 Hz, which matched well with the body or sprung mass and tire or unsprung mass natural frequencies [15,16,17], approximated analytically [14].

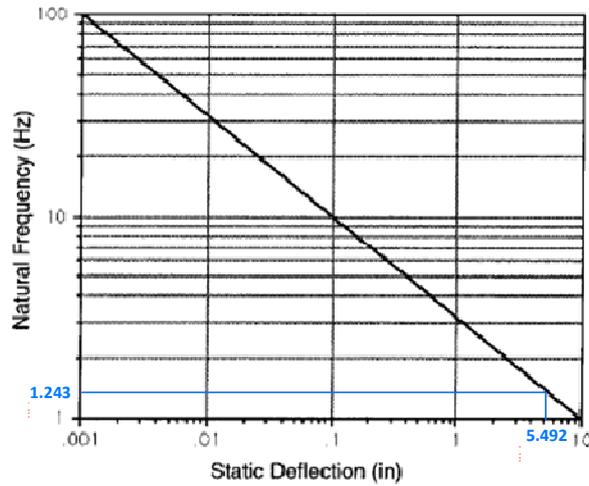
$$\begin{aligned}
 \text{Sprung mass natural frequency}(\omega_{n_s}) &= \sqrt{\frac{\text{Ride Rate}}{M_s}} = \sqrt{\frac{19565.216}{320}} = 7.819\text{rad/sec} \\
 &= 0.159 \times 7.819 = 1.243\text{Hz}
 \end{aligned}
 \tag{3}$$

$$\begin{aligned}
 \text{Unsprung mass natural frequency}(\omega_{n_u}) &= \sqrt{\frac{(k_s + k_u)}{M_u}} = \sqrt{\frac{172500}{37.5}} = 67.822\text{rad/sec} \\
 &= 0.159 \times 67.822 = 10.783\text{Hz}
 \end{aligned}
 \tag{4}$$

The static deflection obtained analytically [14] was also observed to be in a close agreement, as could be seen in Figure 5, with the one that could be read on the standard nomograms [5] representing static deflection against corresponding natural frequencies.



**Figure 3: QFRCo Sprung Mass Transmissibility    Figure 4: QFRCo Unsprung Mass Transmissibility**



**Figure 5: QFRCo Suspension Static Deflection on Nomograms**

The derived model was evaluated further for consistency and reliability by obtaining the simulative responses using vehicle and road parametric values as found in the literature. The input to the vehicle in the research work reported by Senthilkumar [9] was modeled as a discrete step of 0.08 m height, given by Eqn. 5, characterizing the vehicle coming out of a pot hole.

$$x = \begin{cases} 0, & t \leq 1 \\ 0.08, & t > 1 \end{cases} \tag{5}$$

The responses of sprung mass in terms of accelerations and displacements so obtained were found to be in close agreement, as could be seen from Figure 6, upon comparison with those reported in the work by the researcher, thus qualifying the model and approving it for predicting the responses of different vehicle configurations to various deterministic and random input disturbances.

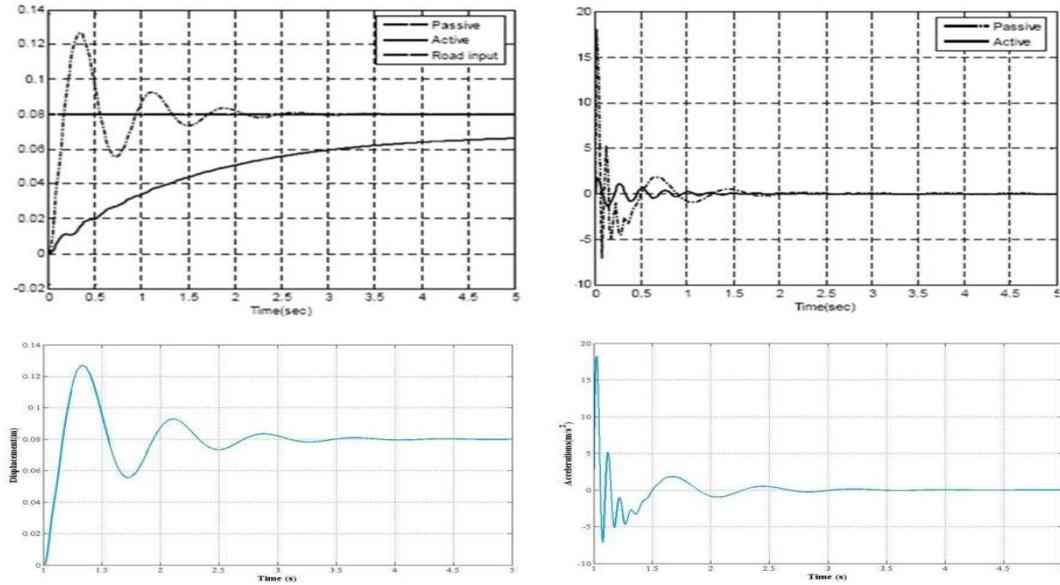


Figure 6: QFRCo Sprung Mass Responses to a Step Input: Validation with Literature

**QFRCo Parametric Analysis**

To understand the influence of suspension parameters, mainly spring stiffness and damping coefficient, a parametric analysis was performed. The performance was evaluated in terms of transmissibilities and vibratory responses to a step input disturbance for different combinations of suspension stiffness and damping coefficients [14]. Shuang et al [18], Anil Shirahatti et al [19] in their respective researches have performed similar analysis of studying the influence of suspension parameters and their sensitivity on the performance. For better interpretation of the effects while varying one particular parameter all others were kept at their respective default values while predicting the responses.

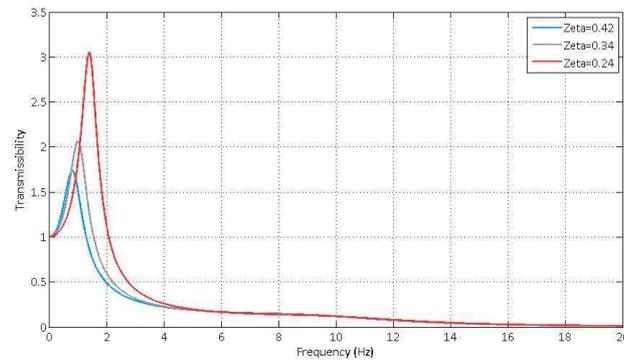


Figure 7: QFRCo Sprung Mass Transmissibility: Effect of Spring Stiffness

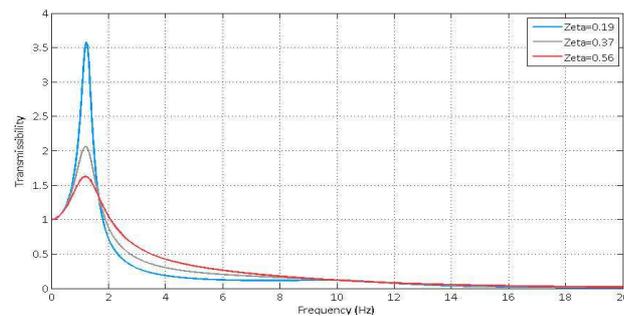
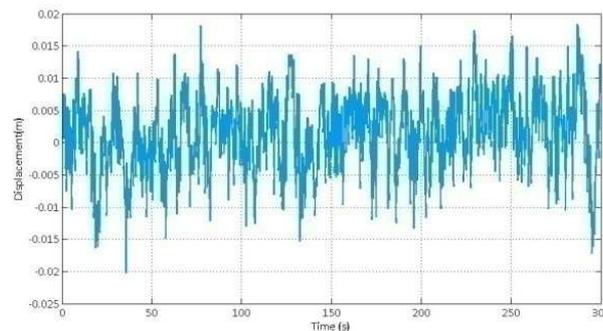


Figure 8: QFRCo Sprung Mass Transmissibility: Effect of Damping Coefficient

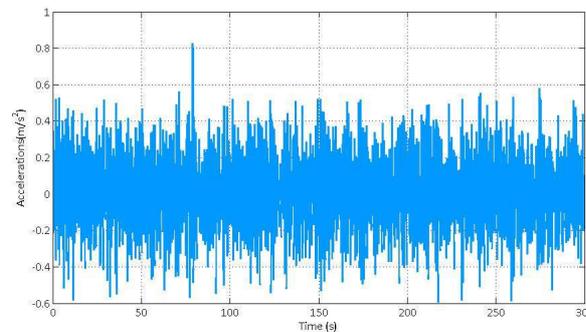
The transmissibility plots for various suspension stiffness and damping coefficients could be seen in Figure 7 and Figure 8 respectively. The typical trend of the plots illustrates graphically an inherent tradeoff between resonance amplitude control and high frequency isolation associated with suspension systems. It could be observed from these plots that low suspension stiffness results in reduced transmissibilities at low frequencies, but the transmissibility amplitudes increase on the higher frequency side, resulting into poor attenuation of the vibrations and a harsh ride. Similarly, at low damping, the resonant transmissibility, corresponding to natural frequencies is relatively large, while the transmissibility at higher frequencies is quite low. As the damping is increased, the resonant peaks are attenuated, but isolation is lost both at high frequency and at frequencies between the two natural frequencies of the system.

### QFRCo: Responses to Random input

The model was evaluated further with a random input C60, representing traveling over an ISO classified grade C road [14] at a speed of 60km/h, and the responses of sprung and unsprung mass were obtained. Figure 9 and Figure 10 depict representative plots of sprung mass displacements and accelerations to the said input RandomC60.

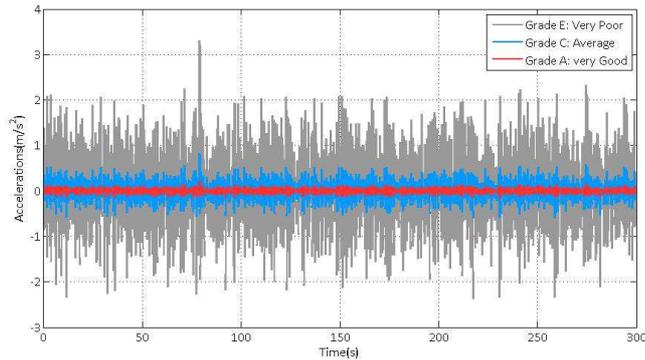


**Figure 9: QFRCo Sprung Mass Displacement: Randomc60**

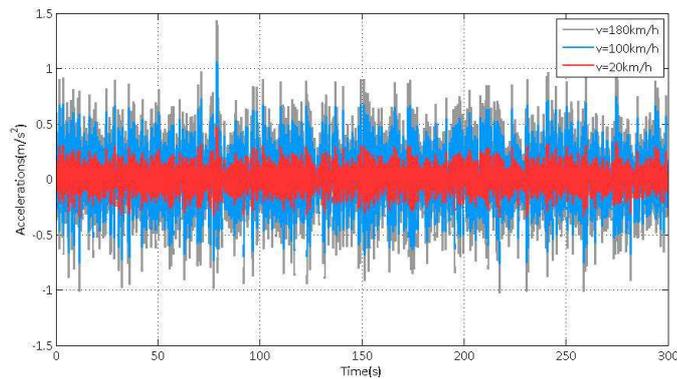


**Figure 10: QFRCo Sprung Mass Accelerations: Randomc60**

The sensitivity of the proposed technique to the influencing parameters was confirmed by employing various combinations of suspension spring stiffness and damping coefficient [18,19] to visualize acceleration responses to a variety of ISO classified road grades [20,21] traversed at different speeds.



**Figure 11: QFRCo Sprung Mass Accelerations: Random: Effect of Road Grades**



**Figure 12: QFRCo Sprung Mass Accelerations: Random: Effect of Speed**

The representative acceleration plots in Figure 11 and Figure 12 persuaded while traversing different ISO classified roads at a constant speed of 60km/h and travelling on the grade C road at different speeds, respectively, demonstrated a trend very much in confirmation with the theoretical anticipation of a compromised comfort due to excessive accelerations on poor road surfaces and at higher speeds.

## CONCLUSIONS

The paper proposes a 2DOF quarter car model of vehicle suspension and delineates its implementation in MATLAB/Simulink for predicting and analyzing its dynamical responses to a variety of input road disturbances modeled both as discrete and random occurrences.

The technique after analytical qualification with the research found in the literature was extended further to perform parametric analysis to investigate effects of influencing parameters, mainly the suspension spring stiffness and damping coefficient, over and above the driving speed and ISO classified road qualities. The results revealed the proposed modeling technique to be sensitive enough to capture the differences in the expected dynamic responses due to vehicular configurations and suspension parameters proving its worth.

The model, though accurate enough for conceptual investigations, requires experimental validation. However because of the simplifications in the model, the results could not be directly compared to experimental ones from field tests. Models with higher complexities could be derived as an extension of the one under consideration for visualizing and analyzing the performance of diverse vehicle configurations under various driving situations, right at the design stage itself, thereby eliminating the need for expensive prototype building and tedious, time consuming testing procedures.

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